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Stable Image Enhancement by Approximated Forward-and-Backward Wave Equation

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Abstract The recently-proposed family of Forward-and-Backward (FAB) Telegraph-Diffusion (TeD) equations [9] is analyzed in the context of image enhancement. Stability of such schemes in terms of energy convergence is investigated. An approximated version of the enhancement operator that offers an increased stability is proposed and examined. This scheme is implemented in both stable and unstable regimes of the original FAB-TeD operator and shown to yield similar results to those obtained by the latter, without losing stability. The theoretical conclusions regarding stability of the approximated FAB-TeD are reinforced by simulations, exhibiting enhanced images with sharpened edges and yet very limited compromise on the quality of other image details.

Keywords Diffusion equations, FAB Telegraph-Diffusion, Stable image enhancement, Image restoration, Wave equations

1 Introduction

One of the most difficult, yet important, tasks in image processing is image enhancement, i.e. improving the visibility of some features in an image that has been affected by blurring and/or contaminated by noise. The difficulties in such enhancement are twofold. Firstly, enhancing meaningful features usually generates additional noise. Secondly, sharpening processes are often unstable and difficult to control, resulting, even in noiseless images, in spurious artifacts.

Recently proposed processing schemes, based on spatially-varying Partial Differential Equations (PDEs), offer an elegant solution to one of these problems. By locally varying the equation coefficients, these methods achieve image-dependent, anisotropic behavior, which allows differential treatments of meaningful features/details and noisy areas [8].

However, enhancement techniques based on PDE-based processing, such as the Forward-and-Backward (FAB) methods ([3], [9]), still remain ill-posed in the continuous settings. The most difficult problem encountered in such processing is that in some cases the energy of the solution drastically increases after arbitrarily short time (here, and in the rest of this paper, unless otherwise specified, 'time' means the independent variable of the evolution of the dynamic process).

This problem was addressed in the discrete space in [13]. There the authors proposed a spatial discretization scheme of the FAB equation that satisfies the minimum-maximum principle, thus preventing explosion of the solution.

Here we adopt a different approach. We do not aim to find a well-posed formulation of the enhancement problem. Instead, we consider a different enhancement problem which is still ill posed, and, as such, the energy of its solution still may explode after long time. However, in this case we require that the solution energy will remain bounded during finite time. This allows achieving enhancement of an image, without introducing artifacts that are characteristic of the unstable regime.

This paper expands the preliminary report presented by the authors in [10], providing more theoretical and experimental results.

2 Enhancement Methods

2.1 Background

The basic image enhancement problem is concerned with finding the signal u given the following smoothed noisy input:

$$\tilde{u} = S(u) + \hat{n},$$

where S is some smoothing kernel which may result, for example, from optical problems, light dispersion or low resolution imaging constraints. The noise $\hat{n}$ is usually assumed to be zero-mean *iid* Gaussian noise.

The above problem requires a model of an "ideal" image in order to derive the enhancement strategy and to evaluate the result. Several such models have been proposed over the years. This paper is concerned mainly with the piecewise-smooth model, where the "ideal" target image consists of smooth areas separated by edges. We first demonstrate this model on a simpler example of image denoising.

Most denoising problems assume $S(u) = u$, therefore, one of the solutions is anisotropic smoothing. There has been some confusion about the use of the term "anisotropic smoothing" in image processing. When the term was introduced in [8], it was meant to describe spatially-varying (but similar in all directions) smoothing, i.e. strong smoothing of flat areas and weak smoothing of edges. The more meaningful "anisotropic" was redefined and used later in [14], where it accounted for directional smoothing. In this paper we focus on the former, historical meaning, applied to image processing by PDEs.

Smoothing properties of PDEs have been used in image processing for the purposes of denoising ([13], [3], [4], [11]) since the introduction of the Perona-Malik (PM) diffusion in [8]. Several variants of these schemes resulted from mathematical analysis of the Total-Variation minimization [11], whereas others were inspired by